4

Centrifugal Compressors and Fans

4.1 INTRODUCTION

This chapter will be concerned with power absorbing turbomachines, used to handle compressible fluids. There are three types of turbomachines: fans, blowers, and compressors. A fan causes only a small rise in stagnation pressure of the flowing fluid. A fan consists of a rotating wheel (called the impeller), which is surrounded by a stationary member known as the housing. Energy is transmitted to the air by the power-driven wheel and a pressure difference is created, providing airflow. The air feed into a fan is called induced draft, while the air exhausted from a fan is called forced draft. In blowers, air is compressed in a series of successive stages and is often led through a diffuser located near the exit. The overall pressure rise may range from 1.5 to 2.5 atm with shaft speeds up to 30,000 rpm or more.

4.2 CENTRIFUGAL COMPRESSOR

The compressor, which can be axial flow, centrifugal flow, or a combination of the two, produces the highly compressed air needed for efficient combustion. In turbocompressors or dynamic compressors, high pressure is achieved by imparting kinetic energy to the air in the impeller, and then this kinetic energy converts into pressure in the diffuser. Velocities of airflow are quite high and the Mach number of the flow may approach unity at many points in the air stream. Compressibility effects may have to be taken into account at every stage of the compressor. Pressure ratios of 4:1 are typical in a single stage, and ratios of 6:1 are possible if materials such as titanium are used. There is renewed interest in the centrifugal stage, used in conjunction with one or more axial stages, for small turbofan and turboprop aircraft engines. The centrifugal compressor is not suitable when the pressure ratio requires the use of more than one stage in series because of aerodynamic problems. Nevertheless, two-stage centrifugal compressors have been used successfully in turbofan engines.

Figure 4.1 shows part of a centrifugal compressor. It consists of a stationary casing containing an impeller, which rotates and imparts kinetic energy to the air and a number of diverging passages in which the air decelerates. The deceleration converts kinetic energy into static pressure. This process is known as diffusion, and the part of the centrifugal compressor containing the diverging passages is known as the diffuser. Centrifugal compressors can be built with a double entry or a single entry impeller. Figure 4.2 shows a double entry centrifugal compressor.

Air enters the impeller eye and is whirled around at high speed by the vanes on the impeller disc. After leaving the impeller, the air passes through a diffuser in which kinetic energy is exchanged with pressure. Energy is imparted to the air by the rotating blades, thereby increasing the static pressure as it moves from eye radius r_1 to tip radius r_2. The remainder of the static pressure rise is achieved in the diffuser. The normal practice is to design the compressor so that about half the pressure rise occurs in the impeller and half in the diffuser. The air leaving the diffuser is collected and delivered to the outlet.

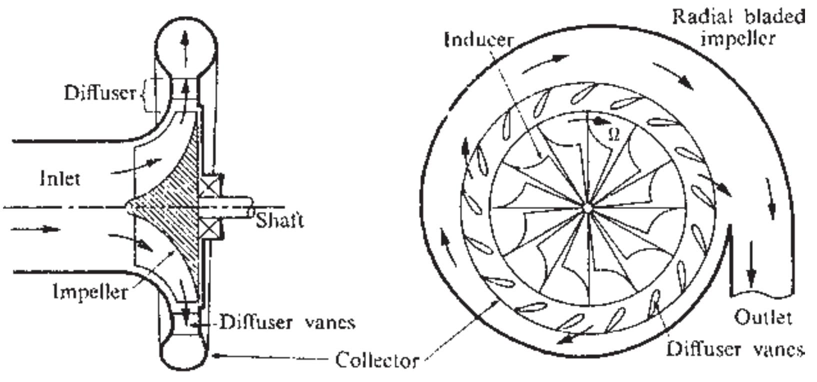

Figure 4.1 Typical centrifugal compressor.

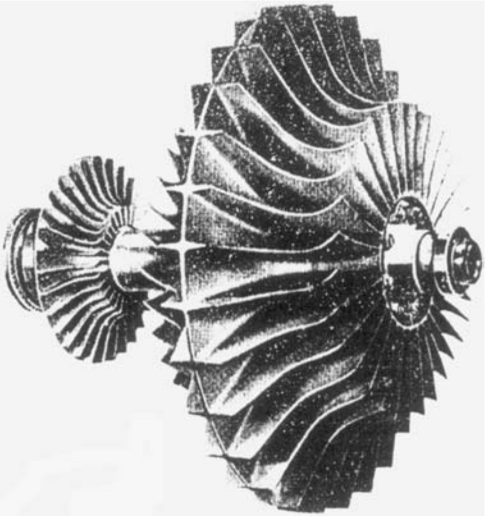

Figure 4.2 Double-entry main stage compressor with side-entry compressor for cooling air. (Courtesy of Rolls-Royce, Ltd.)

4.3 THE EFFECT OF BLADE SHAPE ON PERFORMANCE

As discussed in Chapter 2, there are three types of vanes used in impellers. They are: forward-curved, backward-curved, and radial vanes, as shown in Fig. 4.3.

The impellers tend to undergo high stress forces. Curved blades, such as those used in some fans and hydraulic pumps, tend to straighten out due to centrifugal force and bending stresses are set up in the vanes. The straight radial

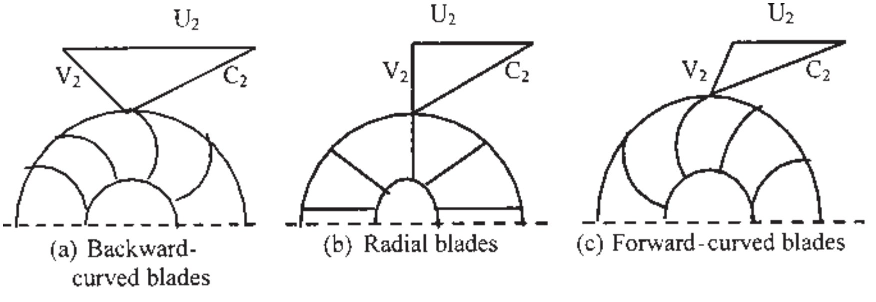

Figure 4.3 Shapes of centrifugal impellar blades: (a) backward-curved blades, (b) radial blades, and (c) forward-curved blades.

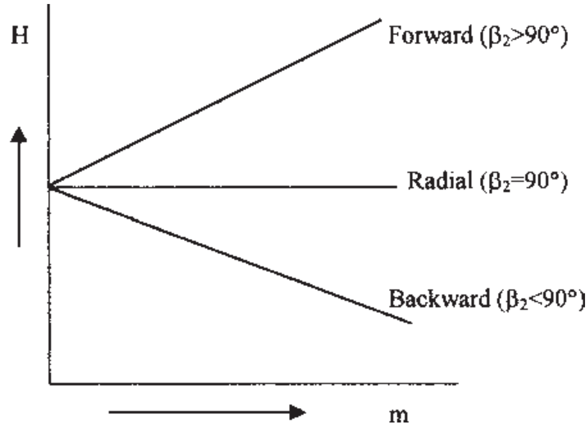

Figure 4.4 Pressure ratio or head versus mass flow or volume flow, for the three blade shapes.

blades are not only free from bending stresses, they may also be somewhat easier to manufacture than curved blades.

Figure 4.3 shows the three types of impeller vanes schematically, along with the velocity triangles in the radial plane for the outlet of each type of vane. Figure 4.4 represents the relative performance of these types of blades. It is clear that increased mass flow decreases the pressure on the backward blade, exerts the same pressure on the radial blade, and increases the pressure on the forward blade. For a given tip speed, the forward-curved blade impeller transfers maximum energy, the radial blade less, and the least energy is transferred by the backward-curved blades. Hence with forward-blade impellers, a given pressure ratio can be achieved from a smaller-sized machine than those with radial or backward-curved blades.

4.4 VELOCITY DIAGRAMS

Figure 4.5 shows the impeller and velocity diagrams at the inlet and outlet. Figure 4.5a represents the velocity triangle when the air enters the impeller in the axial direction. In this case, absolute velocity at the inlet, $C_1 = C_{a1}$. Figure 4.5b represents the velocity triangle at the inlet to the impeller eye and air enters through the inlet guide vanes. Angle θ is made by C_1 and C_{a1} and this angle is known as the angle of prewhirl. The absolute velocity C_1 has a whirl component C_{w1}. In the ideal case, air comes out from the impeller tip after making an angle of 90° (i.e., in the radial direction), so $C_{w2} = U_2$. That is, the whirl component is exactly equal to the impeller tip velocity. Figure 4.5c shows the ideal velocity

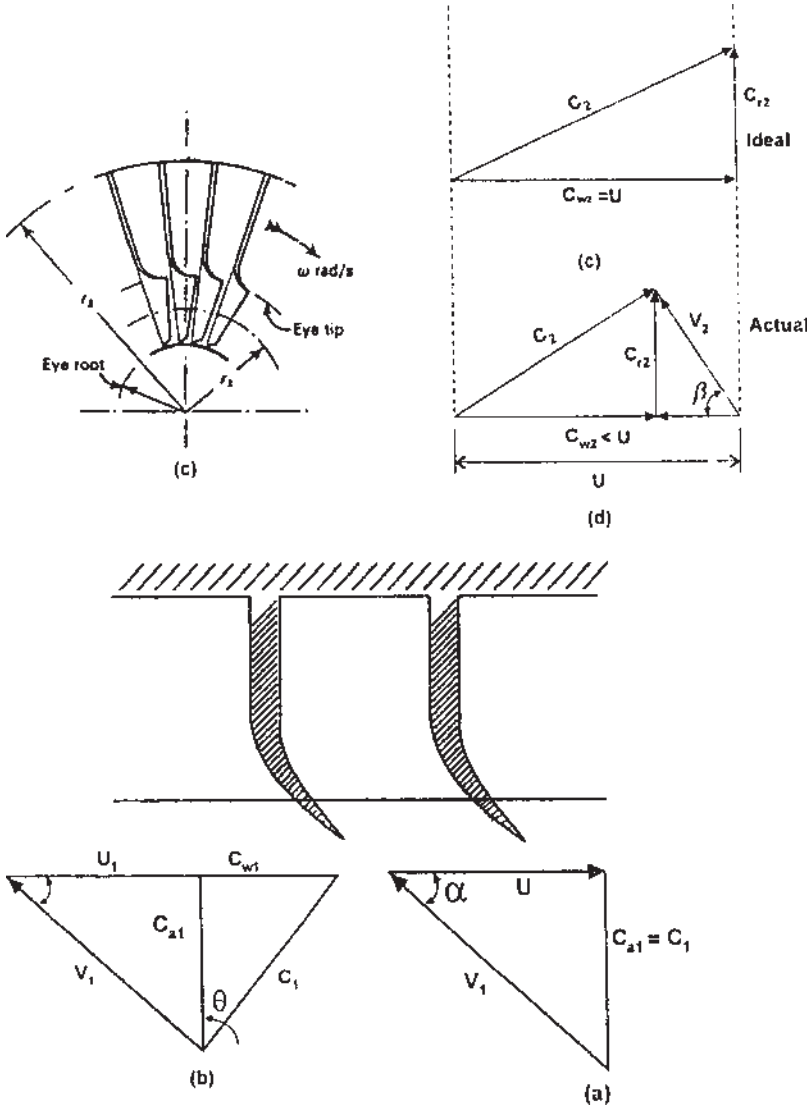

Figure 4.5 Centrifugal impellar and velocity diagrams.

triangle. But there is some slip between the impeller and the fluid, and actual values of C_{w1} are somewhat less than U_2. As we have already noted in the centrifugal pump, this results in a higher static pressure on the leading face of a vane than on the trailing face. Hence, the air is prevented from acquiring a whirl velocity equal to the impeller tip speed. Figure 4.5d represents the actual velocity triangle.

4.5 SLIP FACTOR

From the above discussion, it may be seen that there is no assurance that the actual fluid will follow the blade shape and leave the compressor in a radial direction. Thus, it is convenient to define a slip factor σ as:

$$\sigma = \frac{C_{w2}}{U_2} \tag{4.1}$$

Figure 4.6 shows the phenomenon of fluid slip with respect to a radial blade. In this case, C_{w2} is not equal to U_2; consequently, by the above definition, the slip factor is less than unity. If radial exit velocities are to be achieved by the actual fluid, the exit blade angle must be curved forward about 10–14 degrees. The slip factor is nearly constant for any machine and is related to the number of vanes on the impeller. Various theoretical and empirical studies of the flow in an impeller channel have led to formulas for

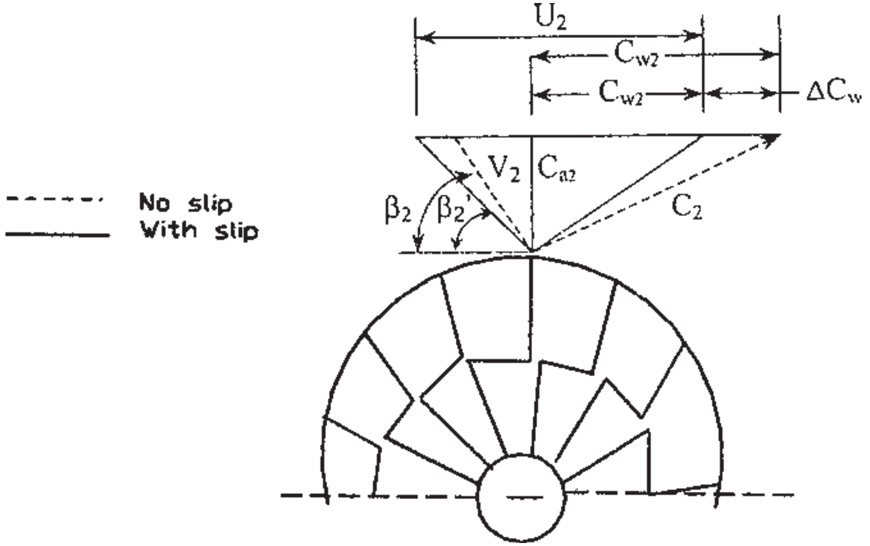

Figure 4.6 Centrifugal compressor impeller with radial vanes.

slip factors: For radial vaned impellers, the formula for σ is given by Stanitz as follows:

$$\sigma = 1 - \frac{0.63\pi}{n} \tag{4.2}$$

where n is the number of vanes. The velocity diagram indicates that C_{w2} approaches U_2 as the slip factor is increased. Increasing the number of vanes may increase the slip factor but this will decrease the flow area at the inlet. A slip factor of about 0.9 is typical for a compressor with 19–21 vanes.

4.6 WORK DONE

The theoretical torque will be equal to the rate of change of angular momentum experienced by the air. Considering a unit mass of air, this torque is given by theoretical torque,

$$\tau = C_{w2} r_2 \tag{4.3}$$

where, C_{w2} is whirl component of C_2 and r_2 is impeller tip radius.

Let ω = angular velocity. Then the theoretical work done on the air may be written as:

Theoretical work done $W_c = C_{w2} r_2 \omega = C_{w2} U_2$.

Using the slip factor, we have theoretical $W_c = \sigma U_2^2$ (treating work done on the air as positive)

In a real fluid, some of the power supplied by the impeller is used in overcoming losses that have a braking effect on the air carried round by the vanes. These include windage, disk friction, and casing friction. To take account of these losses, a power input factor can be introduced. This factor typically takes values between 1.035 and 1.04. Thus the actual work done on the air becomes:

$$W_c = \psi \sigma U_2^2 \tag{4.4}$$

(assuming $C_{w1} = 0$, although this is not always the case.)

Temperature equivalent of work done on the air is given by:

$$T_{02} - T_{01} = \frac{\psi \sigma U_2^2}{C_p}$$

where T_{01} is stagnation temperature at the impeller entrance; T_{02} is stagnation temperature at the impeller exit; and C_p is mean specific heat over this temperature range. As no work is done on the air in the diffuser, $T_{03} = T_{02}$, where T_{03} is the stagnation temperature at the diffuser outlet.

The compressor isentropic efficiency (η_c) may be defined as:

$$\eta_c = \frac{T_{03'} - T_{01}}{T_{03} - T_{01}}$$

(where $T_{03'}$ = isentropic stagnation temperature at the diffuser outlet) or

$$\eta_c = \frac{T_{01}\left(T_{03'}/T_{01} - 1\right)}{T_{03} - T_{01}}$$

Let P_{01} be stagnation pressure at the compressor inlet and; P_{03} is stagnation pressure at the diffuser exit. Then, using the isentropic P–T relationship, we get:

$$\frac{P_{03}}{P_{01}} = \left(\frac{T_{03'}}{T_{01}}\right)^{\gamma/(\gamma-1)} = \left[1 + \frac{\eta_c(T_{03} - T_{01})}{T_{01}}\right]^{\gamma/(\gamma-1)}$$

$$= \left[1 + \frac{\eta_c \psi \sigma U_2{}^2}{C_p T_{01}}\right]^{\gamma/(\gamma-1)} \qquad (4.5)$$

Equation (4.5) indicates that the pressure ratio also depends on the inlet temperature T_{01} and impeller tip speed U_2. Any lowering of the inlet temperature T_{01} will clearly increase the pressure ratio of the compressor for a given work input, but it is not under the control of the designer. The centrifugal stresses in a rotating disc are proportional to the square of the rim. For single sided impellers of light alloy, U_2 is limited to about 460 m/s by the maximum allowable centrifugal stresses in the impeller. Such speeds produce pressure ratios of about 4:1. To avoid disc loading, lower speeds must be used for double-sided impellers.

4.7 DIFFUSER

The designing of an efficient combustion system is easier if the velocity of the air entering the combustion chamber is as low as possible. Typical diffuser outlet velocities are in the region of 90 m/s. The natural tendency of the air in a diffusion process is to break away from the walls of the diverging passage, reverse its direction and flow back in the direction of the pressure gradient, as shown in Fig. 4.7. Eddy formation during air deceleration causes loss by reducing the maximum pressure rise. Therefore, the maximum permissible included angle of the vane diffuser passage is about 11°. Any increase in this angle leads to a loss of efficiency due to

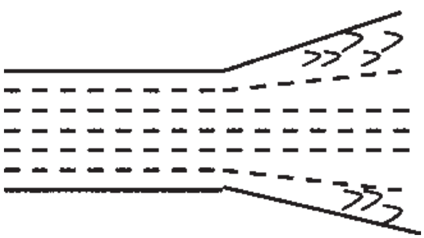

Figure 4.7 Diffusing flow.

boundary layer separation on the passage walls. It should also be noted that any change from the design mass flow and pressure ratio would also result in a loss of efficiency. The use of variable-angle diffuser vanes can control the efficiency loss. The flow theory of diffusion, covered in Chapter 2, is applicable here.

4.8 COMPRESSIBILITY EFFECTS

If the relative velocity of a compressible fluid reaches the speed of sound in the fluid, separation of flow causes excessive pressure losses. As mentioned earlier, diffusion is a very difficult process and there is always a tendency for the flow to break away from the surface, leading to eddy formation and reduced pressure rise. It is necessary to control the Mach number at certain points in the flow to mitigate this problem. The value of the Mach number cannot exceed the value at which shock waves occur. The relative Mach number at the impeller inlet must be less than unity.

As shown in Fig. 4.8a, the air breakaway from the convex face of the curved part of the impeller, and hence the Mach number at this point, will be very important and a shock wave might occur. Now, consider the inlet velocity triangle again (Fig. 4.5b). The relative Mach number at the inlet will be given by:

$$\mathrm{M}_1 = \frac{V_1}{\sqrt{\gamma \mathrm{R} T_1}} \tag{4.6}$$

where T_1 is the static temperature at the inlet.

It is possible to reduce the Mach number by introducing the prewhirl. The prewhirl is given by a set of fixed intake guide vanes preceding the impeller.

As shown in Fig. 4.8b, relative velocity is reduced as indicated by the dotted triangle. One obvious disadvantage of prewhirl is that the work capacity of

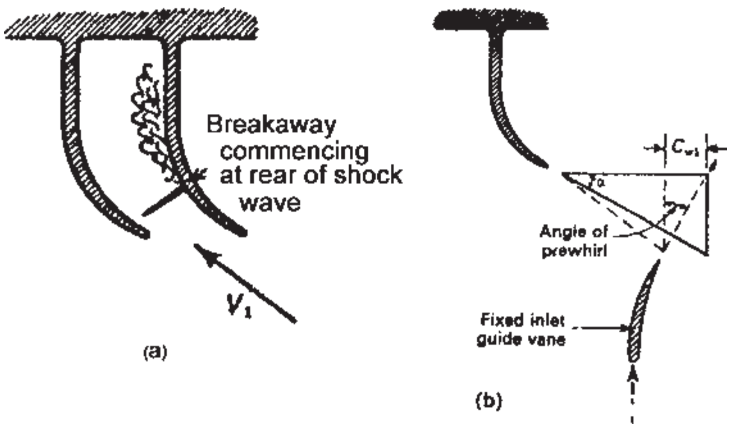

Figure 4.8 a) Breakaway commencing at the aft edge of the shock wave, and b) Compressibility effects.

the compressor is reduced by an amount $U_1 C_{w1}$. It is not necessary to introduce prewhirl down to the hub because the fluid velocity is low in this region due to lower blade speed. The prewhirl is therefore gradually reduced to zero by twisting the inlet guide vanes.

4.9 MACH NUMBER IN THE DIFFUSER

The absolute velocity of the fluid becomes a maximum at the tip of the impeller and so the Mach number may well be in excess of unity. Assuming a perfect gas, the Mach number at the impeller exit M_2 can be written as:

$$M_2 = \frac{C_2}{\sqrt{\gamma R T_2}} \tag{4.7}$$

However, it has been found that as long as the radial velocity component (C_{r2}) is subsonic, Mach number greater than unity can be used at the impeller tip without loss of efficiency. In addition, supersonic diffusion can occur without the formation of shock waves provided constant angular momentum is maintained with vortex motion in the vaneless space. High Mach numbers at the inlet to the diffuser vanes will also cause high pressure at the stagnation points on the diffuser vane tips, which leads to a variation of static pressure around the circumference of the diffuser. This pressure variation is transmitted upstream in a radial direction through the vaneless space and causes cyclic loading of the impeller. This may lead to early fatigue failure when the exciting frequency is of the same order as one of the natural frequencies of the impeller vanes. To overcome this concern, it is a common a practice to use prime numbers for the impeller vanes and an even number for the diffuser vanes.

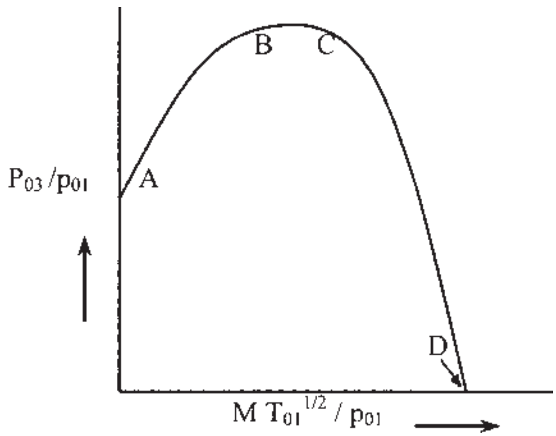

Figure 4.9 The theoretical centrifugal compressor characteristic.

4.10 CENTRIFUGAL COMPRESSOR CHARACTERISTICS

The performance of compressible flow machines is usually described in terms of the groups of variables derived in dimensional analysis (Chapter 1). These characteristics are dependent on other variables such as the conditions of pressure and temperature at the compressor inlet and physical properties of the working fluid. To study the performance of a compressor completely, it is necessary to plot P_{03}/P_{01} against the mass flow parameter $m\frac{\sqrt{T_{01}}}{P_{01}}$ for fixed speed intervals of $\frac{N}{\sqrt{T_{01}}}$. Figure 4.9 shows an idealized fixed speed characteristic. Consider a valve placed in the delivery line of a compressor running at constant speed. First, suppose that the valve is fully closed. Then the pressure ratio will have some value as indicated by Point A. This pressure ratio is available from vanes moving the air about in the impeller. Now, suppose that the valve is opened and airflow begins. The diffuser contributes to the pressure rise, the pressure ratio increases, and at Point B, the maximum pressure occurs. But the compressor efficiency at this pressure ratio will be below the maximum efficiency. Point C indicates the further increase in mass flow, but the pressure has dropped slightly from the maximum possible value. This is the design mass flow rate pressure ratio. Further increases in mass flow will increase the slope of the curve until point D. Point D indicates that the pressure rise is zero. However, the above-described curve is not possible to obtain.

4.11 STALL

Stalling of a stage will be defined as the aerodynamic stall, or the breakaway of the flow from the suction side of the blade airfoil. A multistage compressor may operate stably in the unsurged region with one or more of the stages stalled, and the rest of the stages unstalled. Stall, in general, is characterized by reverse flow near the blade tip, which disrupts the velocity distribution and hence adversely affects the performance of the succeeding stages.

Referring to the cascade of Fig. 4.10, it is supposed that some nonuniformity in the approaching flow or in a blade profile causes blade B to stall. The air now flows onto blade A at an increased angle of incidence due to blockage of channel AB. The blade A then stalls, but the flow on blade C is now at a lower incidence, and blade C may unstall. Therefore the stall may pass along the cascade in the direction of lift on the blades. Rotating stall may lead to vibrations resulting in fatigue failure in other parts of the gas turbine.

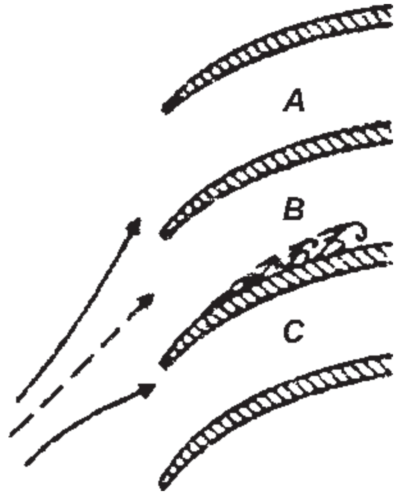

Figure 4.10 Mechanism of stall propagation.

4.12 SURGING

Surging is marked by a complete breakdown of the continuous steady flow throughout the whole compressor, resulting in large fluctuations of flow with time and also in subsequent mechanical damage to the compressor. The phenomenon of surging should not be confused with the stalling of a compressor stage.

Figure 4.11 shows typical overall pressure ratios and efficiencies η_c of a centrifugal compressor stage. The pressure ratio for a given speed, unlike the temperature ratio, is strongly dependent on mass flow rate, since the machine is usually at its peak value for a narrow range of mass flows. When the compressor is running at a particular speed and the discharge is gradually reduced, the pressure ratio will first increase, peaks at a maximum value, and then decreased. The pressure ratio is maximized when the isentropic efficiency has the maximum value. When the discharge is further reduced, the pressure ratio drops due to fall in the isentropic efficiency. If the downstream pressure does not drop quickly there will be backflow accompanied by further decrease in mass flow. In the mean time, if the downstream pressure drops below the compressor outlet pressure, there will be increase in mass flow. This phenomenon of sudden drop in delivery pressure accompanied by pulsating flow is called surging. The point on the curve where surging starts is called the surge point. When the discharge pipe of the compressor is completely choked (mass flow is zero) the pressure ratio will have some value due to the centrifugal head produced by the impeller.

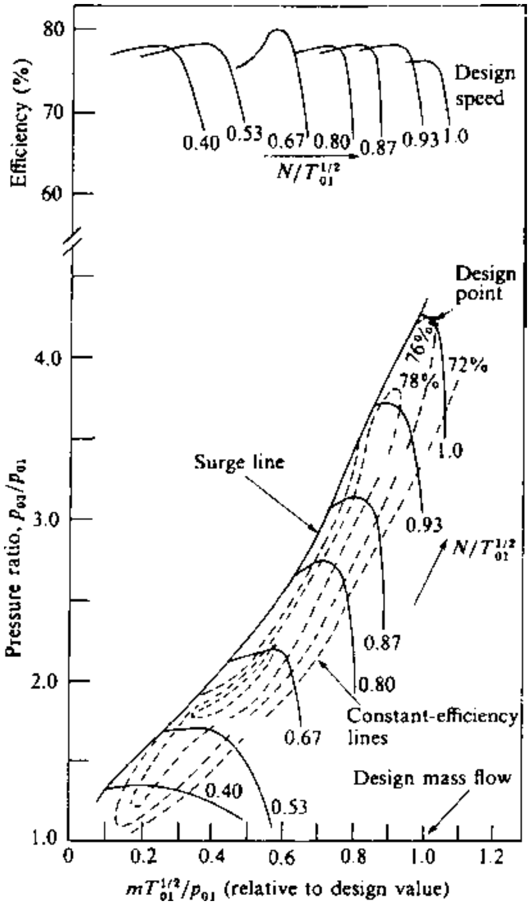

Figure 4.11 Centrifugal compressor characteristics.

Between the zero mass flow and the surge point mass flow, the operation of the compressor will be unstable. The line joining the surge points at different speeds gives the surge line.

4.13 CHOKING

When the velocity of fluid in a passage reaches the speed of sound at any cross-section, the flow becomes choked (air ceases to flow). In the case of inlet flow passages, mass flow is constant. The choking behavior of rotating passages differs from that of the stationary passages, and therefore it is necessary to make separate analysis for impeller and diffuser, assuming one dimensional, adiabatic flow, and that the fluid is a perfect gas.

4.13.1 Inlet

When the flow is choked, $C^2 = a^2 = \gamma RT$. Since $h_0 = h + \frac{1}{2}C^2$, then $C_pT_0 = C_pT + \frac{1}{2}\gamma\,RT$, and

$$\frac{T}{T_0} = \left(1 + \frac{\gamma R}{2C_p}\right)^{-1} = \frac{2}{\gamma + 1} \tag{4.8}$$

Assuming isentropic flow, we have:

$$\left(\frac{\rho}{\rho_0}\right) = \left(\frac{P}{P_0}\right)\left(\frac{T_0}{T}\right) = \left[1 + \frac{1}{2}(\gamma - 1)M^2\right]^{(1-\gamma)/(\gamma-1)} \tag{4.9}$$

and when $C = a$, $M = 1$, so that:

$$\left(\frac{\rho}{\rho_0}\right) = \left[\frac{2}{(\gamma + 1)}\right]^{1/(\gamma-1)} \tag{4.10}$$

Using the continuity equation, $\left(\frac{\dot{m}}{A}\right) = \rho C = \rho\left[\gamma RT\right]^{1/2}$, we have

$$\left(\frac{\dot{m}}{A}\right) = \rho_0 a_0 \left[\frac{2}{\gamma + 1}\right]^{(\gamma+1)/2(\gamma-1)} \tag{4.11}$$

where (ρ_0 and a_0 refer to inlet stagnation conditions, which remain unchanged. The mass flow rate at choking is constant.

4.13.2 Impeller

When choking occurs in the impeller passages, the relative velocity equals the speed of sound at any section. The relative velocity is given by:

$$V^2 = a^2 = \left[\gamma RT\right] \text{ and } T_{01} = T + \left(\frac{\gamma RT}{2C_p}\right) - \frac{U^2}{2C_p}$$

Therefore,

$$\left(\frac{T}{T_{01}}\right) = \left(\frac{2}{\gamma + 1}\right)\left(1 + \frac{U^2}{2C_pT_{01}}\right) \tag{4.12}$$

Using isentropic conditions,

$$\left(\frac{\rho}{\rho_{01}}\right) = \left[\frac{T}{T_{01}}\right]^{1/(\gamma-1)} \text{ and, from the continuity equation:}$$

$$\begin{aligned}
\left(\frac{\dot{m}}{\mathrm{A}}\right) &= \rho_0 a_{01}\left[\frac{T}{T_{01}}\right]^{(\gamma+1)/2(\gamma-1)} \\
&= \rho_{01} a_{01}\left[\left(\frac{2}{\gamma+1}\right)\left(1+\frac{U^2}{2C_{\mathrm{p}}T_{01}}\right)\right]^{(\gamma+1)/2(\gamma-1)} \\
&= \rho_{01} a_{01}\left(\frac{2+(\gamma-1)U^2/a_{01}^2}{\gamma+1}\right)^{(\gamma+1)/2(\gamma-1)}
\end{aligned} \tag{4.13}$$

Equation (4.13) indicates that for rotating passages, mass flow is dependent on the blade speed.

4.13.3 Diffuser

For choking in the diffuser, we use the stagnation conditions for the diffuser and not the inlet. Thus:

$$\left(\frac{\dot{m}}{\mathrm{A}}\right) = \rho_{02} a_{02}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/2(\gamma-1)} \tag{4.14}$$

It is clear that stagnation conditions at the diffuser inlet are dependent on the impeller process.

Illustrative Example 4.1: Air leaving the impeller with radial velocity 110 m/s makes an angle of 25°30′ with the axial direction. The impeller tip speed is 475 m/s. The compressor efficiency is 0.80 and the mechanical efficiency is 0.96. Find the slip factor, overall pressure ratio, and power required to drive the compressor. Neglect power input factor and assume $\gamma = 1.4$, $T_{01} = 298$ K, and the mass flow rate is 3 kg/s.

Solution:

From the velocity triangle (Fig. 4.12),

$$\tan(\beta_2) = \frac{U_2 - C_{w2}}{C_{r2}}$$

$$\tan(25.5°) = \frac{475 - C_{w2}}{110}$$

Therefore, $C_{w2} = 422.54$ m/s.

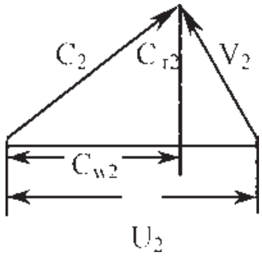

Figure 4.12 Velocity triangle at the impeller tip.

Now, $\sigma = \dfrac{C_{w2}}{U_2} = \dfrac{422.54}{475} = 0.89$

The overall pressure ratio of the compressor:

$$\frac{P_{03}}{P_{01}} = \left[1 + \frac{\eta_c \sigma \psi U_2^2}{C_p T_{01}}\right]^{\gamma/(\gamma-1)} = \left[1 + \frac{(0.80)(0.89)(475^2)}{(1005)(298)}\right]^{3.5} = 4.5$$

The theoretical power required to drive the compressor:

$$P = \left[\frac{m\sigma\psi U_2^2}{1000}\right]\text{kW} = \left[\frac{(3)(0.89)(475^2)}{1000}\right] = 602.42\,\text{kW}$$

Using mechanical efficiency, the actual power required to drive the compressor is: $P = 602.42/0.96 = 627.52$ kW.

Illustrative Example 4.2: The impeller tip speed of a centrifugal compressor is 370 m/s, slip factor is 0.90, and the radial velocity component at the exit is 35 m/s. If the flow area at the exit is 0.18 m^2 and compressor efficiency is 0.88, determine the mass flow rate of air and the absolute Mach number at the impeller tip. Assume air density $=$ 1.57 kg/m^3 and inlet stagnation temperature is 290 K. Neglect the work input factor. Also, find the overall pressure ratio of the compressor.

Solution:

Slip factor: $\sigma = \dfrac{C_{w2}}{U_2}$

Therefore: $C_{w2} = U_2\sigma = (0.90)(370) = 333$ m/s

The absolute velocity at the impeller exit:

$$C_2 = \sqrt{C_{r2}^2 + C_{\omega 2}^2} = \sqrt{333^2 + 35^2} = 334.8\,\text{m/s}$$

The mass flow rate of air: $\dot{m} = \rho_2 A_2 C_{r2} = 1.57{*}0.18{*}35 = 9.89$ kg/s

The temperature equivalent of work done (neglecting ψ):

$$T_{02} - T_{01} = \frac{\sigma U_2^2}{C_p}$$

Therefore, $T_{02} = T_{01} + \dfrac{\sigma U_2^2}{C_p} = 290 + \dfrac{(0.90)(370^2)}{1005} = 412.6\,\text{K}$

The static temperature at the impeller exit,

$$T_2 = T_{02} - \frac{C_2^2}{2C_p} = 412.6 - \frac{334.8^2}{(2)(1005)} = 356.83\,\text{K}$$

The Mach number at the impeller tip:

$$\mathrm{M}_2 = \frac{C_2}{\sqrt{\gamma \mathrm{R} T_2}} = \frac{334.8}{\sqrt{(1.4)(287)(356.83)}} = 0.884$$

The overall pressure ratio of the compressor (neglecting ψ):

$$\frac{P_{03}}{P_{01}} = \left[1 + \frac{\eta_c \sigma \psi U_2^2}{C_p T_{01}}\right]^{3.5} = \left[1 + \frac{(0.88)(0.9)(370^2)}{(1005)(290)}\right]^{3.5} = 3.0$$

Illustrative Example 4.3: A centrifugal compressor is running at 16,000 rpm. The stagnation pressure ratio between the impeller inlet and outlet is 4.2. Air enters the compressor at stagnation temperature of 20°C and 1 bar. If the impeller has radial blades at the exit such that the radial velocity at the exit is 136 m/s and the isentropic efficiency of the compressor is 0.82. Draw the velocity triangle at the exit (Fig. 4.13) of the impeller and calculate slip. Assume axial entrance and rotor diameter at the outlet is 58 cm.

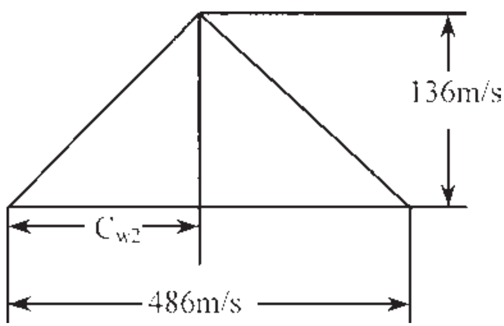

Figure 4.13 Velocity triangle at exit.

Solution:

Impeller tip speed is given by:

$$U_2 = \frac{\pi DN}{60} = \frac{(\pi)(0.58)(16000)}{60} = 486\,\text{m/s}$$

Assuming isentropic flow between impeller inlet and outlet, then

$$T_{02'} = T_{01}(4.2)^{0.286} = 441.69\,\text{K}$$

Using compressor efficiency, the actual temperature rise

$$T_{02} - T_{01} = \frac{\left(T_{02'} - T_{01}\right)}{\eta_c} = \frac{(441.69 - 293)}{0.82} = 181.33\text{K}$$

Since the flow at the inlet is axial, $C_{w1} = 0$

$$W = U_2 C_{w2} = C_p(T_{02} - T_{01}) = 1005(181.33)$$

Therefore: $C_{w2} = \dfrac{1005(181.33)}{486} = 375\,\text{m/s}$

Slip $= 486 - 375 = 111\,\text{m/s}$

Slip factor: $\sigma = \dfrac{C_{w2}}{U_2} = \dfrac{375}{486} = 0.772$

Illustrative Example 4.4: Determine the adiabatic efficiency, temperature of the air at the exit, and the power input of a centrifugal compressor from the following given data:

Impeller tip diameter = 1 m

Speed = 5945 rpm

Mass flow rate of air = 28 kg/s

Static pressure ratio $p_3/p_1 = 2.2$

Atmospheric pressure = 1 bar

Atmospheric temperature = 25°C

Slip factor = 0.90

Neglect the power input factor.

Solution:

The impeller tip speed is given by:

$$U_2 = \frac{\pi DN}{60} = \frac{(\pi)(1)(5945)}{60} = 311\,\text{m/s}$$

The work input: $W = \sigma U_2^2 = \dfrac{(0.9)(311^2)}{1000} = 87\,\text{kJ/kg}$

Using the isentropic P–T relation and denoting isentropic temperature by $T_{3'}$, we get:

$$T_{3'} = T_1\left(\frac{P_3}{P_1}\right)^{0.286} = (298)(2.2)^{0.286} = 373.38\,\mathrm{K}$$

Hence the isentropic temperature rise:

$$T_{3'} - T_1 = 373.38 - 298 = 75.38\,\mathrm{K}$$

The temperature equivalent of work done:

$$T_3 - T_1 = \left(\frac{W}{C_\mathrm{p}}\right) = 87/1.005 = 86.57\,\mathrm{K}$$

The compressor adiabatic efficiency is given by:

$$\eta_\mathrm{c} = \frac{(T_{3'} - T_1)}{(T_3 - T_1)} = \frac{75.38}{86.57} = 0.871 \text{ or } 87.1\%$$

The air temperature at the impeller exit is:

$$T_3 = T_1 + 86.57 = 384.57\,\mathrm{K}$$

Power input:

$$P = \dot{m}W = (28)(87) = 2436\,\mathrm{kW}$$

Illustrative Example 4.5: A centrifugal compressor impeller rotates at 9000 rpm. If the impeller tip diameter is 0.914 m and $\alpha_2 = 20°$, calculate the following for operation in standard sea level atmospheric conditions: (1) U_2, (2) C_{w2}, (3) C_{r2}, (4) β_2, and (5) C_2.

1. Impeller tip speed is given by $U_2 = \frac{\pi DN}{60} = \frac{(\pi)(0.914)(9000)}{60} = 431$ m/s
2. Since the exit is radial and no slip, $C_{w2} = U_2 = 431$ m/s
3. From the velocity triangle,
   $C_{r2} = U_2 \tan(\alpha_2) = (431)\,(0.364) = 156.87$ m/s
4. For radial exit, relative velocity is exactly perpendicular to rotational velocity U_2. Thus the angle β_2 is 90° for radial exit.
5. Using the velocity triangle (Fig. 4.14),

$$C_2 = \sqrt{U_2^2 + C_{r2}^2} = \sqrt{431^2 + 156.87^2} = 458.67\,\mathrm{m/s}$$

Illustrative Example 4.6: A centrifugal compressor operates with no prewhirl is run with a rotor tip speed of 457 m/s. If C_{w2} is 95% of U_2 and $\eta_\mathrm{c} = 0.88$,

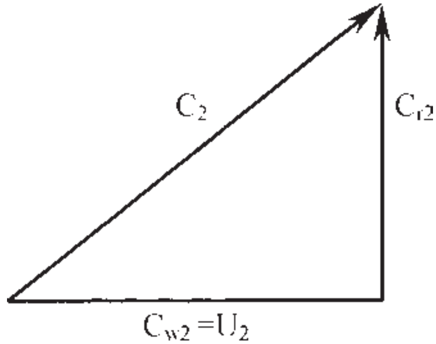

Figure 4.14 Velocity triangle at impeller exit.

calculate the following for operation in standard sea level air: (1) pressure ratio, (2) work input per kg of air, and (3) the power required for a flow of 29 k/s.

Solution:

1. The pressure ratio is given by (assuming $\sigma = \psi = 1$):

$$\frac{P_{03}}{P_{01}} = \left[1 + \frac{\eta_c \sigma \psi U_2^2}{C_p T_{01}}\right]^{\gamma/(\gamma-1)}$$

$$= \left[1 + \frac{(0.88)(0.95)(457^2)}{(1005)(288)}\right]^{3.5} = 5.22$$

2. The work per kg of air

$$W = U_2 C_{w2} = (457)(0.95)(457) = 198.4\,\text{kJ/kg}$$

3. The power for 29 kg/s of air

$$P = \dot{m}W = (29)(198.4) = 5753.6\,\text{kW}$$

Illustrative Example 4.7: A centrifugal compressor is running at 10,000 rpm and air enters in the axial direction. The inlet stagnation temperature of air is 290 K and at the exit from the impeller tip the stagnation temperature is 440 K. The isentropic efficiency of the compressor is 0.85, work input factor $\psi = 1.04$, and the slip factor $\sigma = 0.88$. Calculate the impeller tip diameter, overall pressure ratio, and power required to drive the compressor per unit mass flow rate of air.

Solution:

Temperature equivalent of work done:

$$T_{02} - T_{01} = \frac{\sigma\psi U_2^2}{C_p} \text{ or } \frac{(0.88)(1.04)(U_2^2)}{1005}.$$

Therefore, $U_2 = 405.85\,\text{m/s}$

and $D = \dfrac{60U_2}{\pi N} = \dfrac{(60)(405.85)}{(\pi)(10,000)} = 0.775\,\text{m}$

The overall pressure ratio is given by:

$$\frac{P_{03}}{P_{01}} = \left[1 + \frac{\eta_c \sigma\psi U_2^2}{C_p T_{01}}\right]^{\gamma/(\gamma-1)}$$

$$= \left[1 + \frac{(0.85)(0.88)(1.04)(405.85^2)}{(1005)(290)}\right]^{3.5} = 3.58$$

Power required to drive the compressor per unit mass flow:

$$P = m\psi\sigma U_2^2 = \frac{(1)(0.88)(1.04)(405.85^2)}{1000} = 150.75\,\text{kW}$$

Design Example 4.8: Air enters axially in a centrifugal compressor at a stagnation temperature of 20°C and is compressed from 1 to 4.5 bars. The impeller has 19 radial vanes and rotates at 17,000 rpm. Isentropic efficiency of the compressor is 0.84 and the work input factor is 1.04. Determine the overall diameter of the impeller and the power required to drive the compressor when the mass flow is 2.5 kg/s.

Solution:

Since the vanes are radial, using the Stanitz formula to find the slip factor:

$$\sigma = 1 - \frac{0.63\pi}{n} = 1 - \frac{0.63\pi}{19} = 0.8958$$

The overall pressure ratio

$$\frac{P_{03}}{P_{01}} = \left[1 + \frac{\eta_c \sigma\psi U_2^2}{C_p T_{01}}\right]^{\gamma/(\gamma-1)}, \text{ or } 4.5$$

$$= \left[1 + \frac{(0.84)(0.8958)(1.04)(U_2^2)}{(1005)(293)}\right]^{3.5}, \text{ so } U_2 = 449.9\,\text{m/s}$$

The impeller diameter, $D = \dfrac{60U_2}{\pi N} = \dfrac{(60)(449.9)}{\pi(17,000)} = 0.5053\,\text{m} = 50.53\,\text{cm}.$

The work done on the air $W = \frac{\psi \sigma U_2^2}{1000} = \frac{(0.8958)(1.04)(449.9^2)}{1000} =$ 188.57 kJ/kg

Power required to drive the compressor: $P = \dot{m}W = (2.5)(188.57) =$ 471.43 kW

Design Example 4.9: Repeat problem 4.8, assuming the air density at the impeller tip is 1.8 kg/m^3 and the axial width at the entrance to the diffuser is 12 mm. Determine the radial velocity at the impeller exit and the absolute Mach number at the impeller tip.

Solution:
Slip factor: $\sigma = \frac{C_{w2}}{U_2}$, or $C_{w2} = (0.8958)(449.9) = 403$ m/s
Using the continuity equation,

$$\dot{m} = \rho_2 A_2 C_{r2} = \rho_2 2\pi r_2 b_2 C_{r2}$$

where:

b_2 = axial width
r_2 = radius

Therefore:

$$C_{r2} = \frac{2.5}{(1.8)(2\pi)(0.25)(0.012)} = 73.65 \text{ m/s}$$

Absolute velocity at the impeller exit

$$C_2 = \sqrt{C_{r2}^2 + C_{w2}^2} = \sqrt{73.65^2 + 403^2} = 409.67 \text{ m/s}$$

The temperature equivalent of work done:

$$T_{02} - T_{01} = 188.57/C_p = 188.57/1.005 = 187.63 \text{ K}$$

Therefore, $T_{02} = 293 + 187.63 = 480.63$ K
Hence the static temperature at the impeller exit is:

$$T_2 = T_{02} - \frac{C_2^2}{2C_p} = 480.63 - \frac{409.67^2}{(2)(1005)} = 397 \text{ K}$$

Now, the Mach number at the impeller exit is:

$$M_2 = \frac{C_2}{\sqrt{\gamma R T_2}} = \frac{409.67}{\sqrt{(1.4)(287)(397)}} = 1.03$$

Design Example 4.10: A centrifugal compressor is required to deliver 8 kg/s of air with a stagnation pressure ratio of 4 rotating at 15,000 rpm. The air enters the compressor at 25°C and 1 bar. Assume that the air enters axially with velocity of 145 m/s and the slip factor is 0.89. If the compressor isentropic efficiency is 0.89, find the rise in stagnation temperature, impeller tip speed, diameter, work input, and area at the impeller eye.

Solution:

Inlet stagnation temperature:

$$T_{01} = T_a + \frac{C_1^2}{2C_p} = 298 + \frac{145^2}{(2)(1005)} = 308.46\,\text{K}$$

Using the isentropic P–T relation for the compression process,

$$T_{03'} = T_{01}\left(\frac{P_{03}}{P_{01}}\right)^{(\gamma-1)/\gamma} = (308.46)(4)^{0.286} = 458.55K$$

Using the compressor efficiency,

$$T_{02} - T_{01} = \frac{\left(T_{02'} - T_{01}\right)}{\eta_c} = \frac{(458.55 - 308.46)}{0.89} = 168.64\,\text{K}$$

Hence, work done on the air is given by:

$$W = C_p(T_{02} - T_{01}) = (1.005)(168.64) = 169.48\,\text{kJ/kg}$$

But,

$$W = \sigma U_2^2 = \frac{(0.89)(U_2)}{1000}, \text{ or :}169.48 = 0.89U_2^2/1000$$

or:

$$U_2 = \sqrt{\frac{(1000)(169.48)}{0.89}} = 436.38\,\text{m/s}$$

Hence, the impeller tip diameter

$$D = \frac{60U_2}{\pi N} = \frac{(60)(436.38)}{\pi(15{,}000)} = 0.555\,\text{m}$$

The air density at the impeller eye is given by:

$$\rho_1 = \frac{\text{P}_1}{\text{R}T_1} = \frac{(1)(100)}{(0.287)(298)} = 1.17\,\text{kg/m}^3$$

Using the continuity equation in order to find the area at the impeller eye,

$$A_1 = \frac{\dot{m}}{\rho_1 C_1} = \frac{8}{(1.17)(145)} = 0.047\,\text{m}^2$$

The power input is:

$$P = \dot{m}\ W = (8)(169.48) = 1355.24\,\text{kW}$$

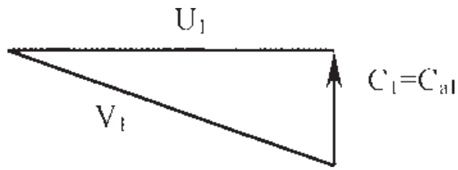

Figure 4.15 The velocity triangle at the impeller eye.

Design Example 4.11: The following data apply to a double-sided centrifugal compressor (Fig. 4.15):

Impeller eye tip diameter:	0.28 m
Impeller eye root diameter:	0.14 m
Impeller tip diameter:	0.48 m
Mass flow of air:	10 kg/s
Inlet stagnation temperature:	290 K
Inlet stagnation pressure:	1 bar
Air enters axially with velocity:	145 m/s
Slip factor:	0.89
Power input factor:	1.03
Rotational speed:	15, 000 rpm

Calculate (1) the impeller vane angles at the eye tip and eye root, (2) power input, and (3) the maximum Mach number at the eye.

Solution:

(1) Let U_{er} be the impeller speed at the eye root. Then the vane angle at the eye root is:

$$\alpha_{er} = \tan^{-1}\left(\frac{C_a}{U_{er}}\right)$$

and

$$U_{er} = \frac{\pi D_{er} N}{60} = \frac{\pi(0.14)(15,000)}{60} = 110\,\text{m/s}$$

Hence, the vane angle at the impeller eye root:

$$\alpha_{er} = \tan^{-1}\left(\frac{C_a}{U_{er}}\right) = \tan^{-1}\left(\frac{145}{110}\right) = 52^{\circ}48'$$

Impeller velocity at the eye tip:

$$U_{\text{et}} = \frac{\pi D_{\text{et}} N}{60} = \frac{\pi(0.28)(15,000)}{60} = 220\,\text{m/s}$$

Therefore vane angle at the eye tip:

$$\alpha_{\text{et}} = \tan^{-1}\left(\frac{C_{\text{a}}}{U_{\text{et}}}\right) = \tan^{-1}\left(\frac{145}{220}\right) = 33^{\circ}23'$$

(2) Work input:

$$W = \dot{m}\psi\sigma U_2^2 = (10)(0.819)(1.03U_2^2)$$

but:

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi(0.48)(15,000)}{60} = 377.14\,\text{m/s}$$

Hence,

$$W = \frac{(10)(0.89)(1.03)(377.14^2)}{1000} = 1303.86\,\text{kW}$$

(3) The relative velocity at the eye tip:

$$V_1 = \sqrt{U_{\text{et}}^2 + C_{\text{a}}^2} = \sqrt{220^2 + 145^2} = 263.5\,\text{m/s}$$

Hence, the maximum relative Mach number at the eye tip:

$$\text{M}_1 = \frac{V_1}{\sqrt{\gamma \text{R} T_1}},$$

where T_1 is the static temperature at the inlet

$$T_1 = T_{01} - \frac{C_1^2}{2C_{\text{p}}} = 290 - \frac{145^2}{(2)(1005)} = 279.54\,\text{K}$$

The Mach number at the inlet then is:

$$\text{M}_1 = \frac{V_1}{\sqrt{\gamma \text{R} T_1}} = \frac{263/5}{\sqrt{(1.4)(287)(279.54)}} = 0.786$$

Design Example 4.12: Recalculate the maximum Mach number at the impeller eye for the same data as in the previous question, assuming prewhirl angle of 20°.

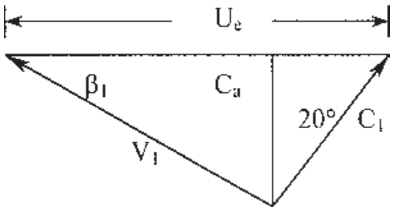

Figure 4.16 The velocity triangle at the impeller eye.

Solution:
Figure 4.16 shows the velocity triangle with the prewhirl angle.
From the velocity triangle:

$$C_1 = \frac{145}{\cos(20^\circ)} = 154.305\,\text{m/s}$$

Equivalent dynamic temperature:

$$\frac{C_1^2}{2C_p} = \frac{154.305^2}{(2)(1005)} = 11.846\,\text{K}$$

$$C_{w1} = \tan(20^\circ)\,C_{a1} = (0.36)(145) = 52.78\,\text{m/s}$$

Relative velocity at the inlet:

$$V_1^2 = C_a^2 + (U_e - C_{w1})^2 = 145^2 + (220 - 52.78)^2, \text{ or } V_1$$

$$= 221.3\,\text{m/s}$$

Therefore the static temperature at the inlet:

$$T_1 = T_{01} - \frac{C_1^2}{2C_p} = 290 - 11.846 = 278.2\,\text{K}$$

Hence,

$$\text{M}_1 = \frac{V_1}{\sqrt{\gamma R T_1}} = \frac{221.3}{\sqrt{(1.4)(287)(278.2)}} = 0.662$$

Note the reduction in Mach number due to prewhirl.

Design Example 4.13: The following data refers to a single-sided centrifugal compressor:

Ambient Temperature:	288 K
Ambient Pressure:	1 bar
Hub diameter:	0.125 m
Eye tip diameter:	0.25 m
Mass flow:	5.5 kg/s
Speed:	16, 500 rpm

Assume zero whirl at the inlet and no losses in the intake duct. Calculate the blade inlet angle at the root and tip and the Mach number at the eye tip.

Solution:

Let: r_h = hub radius

r_t = tip radius

The flow area of the impeller inlet annulus is:

$$A_1 = \pi\left(r_t^2 - r_h^2\right) = \pi\left(0.125^2 - 0.0625^2\right) = 0.038\,\mathrm{m}^2$$

Axial velocity can be determined from the continuity equation but since the inlet density (ρ_1) is unknown a trial and error method must be followed. Assuming a density based on the inlet stagnation condition,

$$\rho_1 = \frac{P_{01}}{\mathrm{R}T_{01}} = \frac{(1)(10^5)}{(287)(288)} = 1.21\,\mathrm{kg/m^3}$$

Using the continuity equation,

$$C_a = \frac{\dot{m}}{\rho_1 A_1} = \frac{5.5}{(1.21)(0.038)} = 119.6\,\mathrm{m/s}$$

Since the whirl component at the inlet is zero, the absolute velocity at the inlet is $C_1 = C_a$.

The temperature equivalent of the velocity is:

$$\frac{C_1^2}{2C_p} = \frac{119.6^2}{(2)(1005)} = 7.12\,\mathrm{K}$$

Therefore:

$$T_1 = T_{01} - \frac{C_1^2}{2C_p} = 288 - 7.12 = 280.9\,\mathrm{K}$$

Using isentropic P–T relationship,

$$\frac{P_1}{P_{01}} = \left(\frac{T_1}{T_{01}}\right)^{\gamma/(\gamma-1)}, \text{ or } P_1 = 10^5\left(\frac{280.9}{288}\right)^{3.5} = 92\,\text{kPa}$$

and:

$$\rho_1 = \frac{P_1}{RT_1} = \frac{(92)(10^3)}{(287)(280.9)} = 1.14\,\text{kg/m}^3, \text{ and}$$

$$C_a = \frac{5.5}{(1.14)(0.038)} = 126.96\,\text{m/s}$$

Therefore:

$$\frac{C_1^2}{2C_p} = \frac{(126.96)^2}{2(1005)} = 8.02\,\text{K}$$

$$T_1 = 288 - 8.02 = 279.98^\circ\,\text{K}$$

$$P_1 = 10^5\left(\frac{279.98}{288}\right)^{3.5} = 90.58\,\text{kPa}$$

$$\rho_1 = \frac{(90.58)(10^3)}{(287)(279.98)} = 1.13\,\text{kg/m}^3$$

Further iterations are not required and the value of $\rho_1 = 1.13\,\text{kg/m}^3$ may be taken as the inlet density and $C_a = C_1$ as the inlet velocity. At the eye tip:

$$U_{et} = \frac{2\pi r_{et} N}{60} = \frac{2\pi(0.125)(16,500)}{60} = 216\,\text{m/s}$$

The blade angle at the eye tip:

$$\beta_{et} = \tan^{-1}\left(\frac{U_{et}}{C_a}\right) = \tan^{-1}\left(\frac{216}{126.96}\right) = 59.56^\circ$$

At the hub,

$$U_{eh} = \frac{2\pi(0.0625)(16,500)}{60} = 108\,\text{m/s}$$

The blade angle at the hub:

$$\beta_{eh} = \tan^{-1}\left(\frac{108}{126.96}\right) = 40.39^\circ$$

The Mach number based on the relative velocity at the eye tip using the inlet velocity triangle is:

$$V_1 = \sqrt{C_a^2 + U_1^2} = \sqrt{126.96^2 + 216^2}, \text{ or } V_1 = 250.6\,\text{m/s}$$

The relative Mach number

$$M = \frac{V_1}{\sqrt{\gamma R T_1}} = \frac{250.6}{\sqrt{(1.4)(287)(279.98)}} = 0.747$$

Design Example 4.14: A centrifugal compressor compresses air at ambient temperature and pressure of 288 K and 1 bar respectively. The impeller tip speed is 364 m/s, the radial velocity at the exit from the impeller is 28 m/s, and the slip factor is 0.89. Calculate the Mach number of the flow at the impeller tip. If the impeller total-to-total efficiency is 0.88 and the flow area from the impeller is $0.085\,\text{m}^2$, calculate the mass flow rate of air. Assume an axial entrance at the impeller eye and radial blades.

Solution:
The absolute Mach number of the air at the impeller tip is:

$$M_2 = \frac{C_2}{\sqrt{\gamma R T_2}}$$

where T_2 is the static temperature at the impeller tip. Let us first calculate C_2 and T_2.
Slip factor:

$$\sigma = \frac{C_{w2}}{U_2}$$

Or:

$$C_{w2} = \sigma U_2 = (0.89)(364) = 323.96\,\text{m/s}$$

From the velocity triangle,

$$C_2^2 = C_{r2}^2 + C_{w2}^2 = 28^2 + 323.96^2 = (1.06)(10^5)\,\text{m}^2/\text{s}^2$$

With zero whirl at the inlet

$$\frac{W}{m} = \sigma U_2^2 = C_p(T_{02} - T_{01})$$

Hence,

$$T_{02} = T_{01} + \frac{\sigma U_2^2}{C_p} = 288 + \frac{(0.89)(364^2)}{1005} = 405.33\,\text{K}$$

Static Temperature

$$T_2 = T_{02} - \frac{C_2^2}{2C_p} = 405.33 - \frac{106000}{(2)(1005)} = 352.6\,\text{K}$$

Therefore,

$$M_2 = \left(\frac{(1.06)(10^5)}{(1.4)(287)(352.6)}\right)^{\frac{1}{2}} = 0.865$$

Using the isentropic P–T relation:

$$\left(\frac{P_{02}}{P_{01}}\right) = \left[1 + \eta_c\left(\frac{T_{02}}{T_{01}} - 1\right)\right]^{\gamma/(\gamma-1)}$$

$$= \left[1 + 0.88\left(\frac{405.33}{288} - 1\right)\right]^{3.5} = 2.922$$

$$\left(\frac{P_2}{P_{02}}\right) = \left(\frac{T_2}{T_{02}}\right)^{3.5} = \left(\frac{352.6}{405.33}\right)^{3.5} = 0.614$$

Therefore,

$$P_2 = \left(\frac{P_2}{P_{02}}\right)\left(\frac{P_{02}}{P_{01}}\right)P_{01}$$

$$= (0.614)(2.922)(1)(100)$$

$$= 179.4\,\text{kPa}$$

$$\rho_2 = \frac{179.4(1000)}{287(352.6)} = 1.773\,\text{kg/m}^3$$

Mass flow:

$$\dot{m} = (1.773)(0.085)(28) = 4.22\,\text{kg/s}$$

Design Example 4.15: The impeller of a centrifugal compressor rotates at 15,500 rpm, inlet stagnation temperature of air is 290 K, and stagnation pressure at inlet is 101 kPa. The isentropic efficiency of impeller is 0.88, diameter of the impellar is 0.56 m, axial depth of the vaneless space is 38 mm, and width of the vaneless space is 43 mm. Assume ship factor as 0.9, power input factor 1.04, mass flow rate as 16 kg/s. Calculate

1. Stagnation conditions at the impeller outlet, assume no fore whirl at the inlet,
2. Assume axial velocity approximately equal to 105 m/s at the impeller outlet, calculate the Mach number and air angle at the impeller outlet,

3. The angle of the diffuser vane leading edges and the Mach number at this radius if the diffusion in the vaneless space is isentropic.

Solution:

1. Impeller tip speed

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.56 \times 15500}{60}$$

$$U_2 = 454.67\ \mathrm{m/s}$$

Overall stagnation temperature rise

$$T_{03} - T_{01} = \frac{\psi \sigma U_2^2}{1005} = \frac{1.04 \times 0.9 \times 454.67^2}{1005}$$
$$= 192.53\mathrm{K}$$

Since $T_{03} = T_{02}$
Therefore, $T_{02} - T_{01} = 192.53\mathrm{K}$ and $T_{02} = 192.53 + 290 = 482.53\mathrm{K}$
Now pressure ratio for impeller

$$\frac{p_{02}}{p_{01}} = \left(\frac{T_{02}}{T_{01}}\right)^{3.5} = \left(\frac{482.53}{290}\right)^{3.5} = 5.94$$

then, $p_{02} = 5.94 \times 101 = 600$ KPa

2.

$$\sigma = \frac{C_{w2}}{U_2}$$

$$C_{w2} = \sigma U_2$$

or

$$C_{w2} = 0.9 \times 454.67 = 409\ \mathrm{m/s}$$

Let $C_{r2} = 105$ m/s
Outlet area normal to periphery

$$A_2 = \pi D_2 \times \text{impeller depth}$$
$$= \pi \times 0.56 \times 0.038$$
$$A_2 = 0.0669\ \mathrm{m}^2$$

From outlet velocity triangle

$$C_2{}^2 = C_{r2}^2 + C_{w2}^2$$
$$= 105^2 + 409^2$$
$$C_2{}^2 = 178306$$

i.e. $C_2 = 422.26\,\text{m/s}$

$$T_2 = T_{02} - \frac{C_2^2}{2C_p} = 482.53 - \frac{422.26^2}{2 \times 1005}$$

$$T_2 = 393.82\,\text{K}$$

Using isentropic P–T relations

$$P_2 = P_{02}\left(\frac{T_2}{T_{02}}\right)^{\frac{\gamma}{\gamma-1}} = 600\left(\frac{393.82}{482.53}\right)^{3.5} = 294.69\,\text{kPa}$$

From equation of state

$$\rho_2 = \frac{P_2}{RT_2} = \frac{293.69 \times 10^3}{287 \times 393.82} = 2.61\,\text{kg/m}^3$$

The equation of continuity gives

$$C_{r2} = \frac{\dot{m}}{A_2 P_2} = \frac{16}{0.0669 \times 2.61} = 91.63\,\text{m/s}$$

Thus, impeller outlet radial velocity = 91.63 m/s
Impeller outlet Mach number

$$M_2 = \frac{C_2}{\sqrt{\gamma R T_2}} = \frac{422.26}{(1.4 \times 287 \times 393.82)^{0.5}}$$

$$M_2 = 1.06$$

From outlet velocity triangle

$$\mathrm{Cos}\,\alpha_2 = \frac{C_{r2}}{C_2} = \frac{91.63}{422.26} = 0.217$$

i.e., $\alpha_2 = 77.47°$

3. Assuming free vortex flow in the vaneless space and for convenience denoting conditions at the diffuser vane without a subscript ($r = 0.28 + 0.043 = 0.323$)

$$C_w = \frac{C_{w2} r_2}{r} = \frac{409 \times 0.28}{0.323}$$

$$C_w = 354.55\,\text{m/s}$$

The radial component of velocity can be found by trial and error. Choose as a first try, $C_r = 105$ m/s

$$\frac{C^2}{2C_p} = \frac{105^2 + 354.55^2}{2 \times 1005} = 68\,\text{K}$$

$T = 482.53 - 68$ (since $T = T_{02}$ in vaneless space)

$$T = 414.53\text{K}$$

$$p = p_{02}\left(\frac{T_2}{T_{02}}\right)^{3.5} = 600\left(\frac{419.53}{482.53}\right)^{3.5} = 352.58\,\text{kPa}$$

$$\rho = \frac{p_2}{\text{R}T_2} = \frac{294.69}{287 \times 393.82}$$

$$\rho = 2.61\,\text{kg/m}^3$$

The equation of continuity gives

$$\begin{aligned} A &= 2\pi r \times \text{depth of vanes} \\ &= 2\pi \times 0.323 \times 0.038 \\ &= 0.0772\,\text{m}^2 \end{aligned}$$

$$C_r = \frac{16}{2.61 \times 0.0772} = 79.41\text{ m/s}$$

Next try $C_r = 79.41$ m/s

$$\frac{C^2}{2C_p} = \frac{79.41^2 + 354.55^2}{2 \times 1005} = 65.68$$

$$T = 482.53 - 65.68 = 416.85\text{K}$$

$$p = p_{02}\left(\frac{T}{T_{02}}\right)^{3.5} = 600\left(\frac{416.85}{482.53}\right)^{3.5}$$

$$p = 359.54\,\text{Pa}$$

$$\rho = \frac{359.54}{416.85 \times 287} = 3\,\text{kg/m}^3$$

$$C_r = \frac{16}{3.0 \times 0.772} = 69.08 \text{ m/s}$$

Try $C_r = 69.08$ m/s

$$\frac{C^2}{2C_p} = \frac{69.08^2 + 354.55^2}{2 \times 1005} = 64.9$$

$$T = 482.53 - 64.9 = 417.63 \text{ K}$$

$$p = p_{02}\left(\frac{T}{T_{02}}\right)^{3.5} = 600\left(\frac{417.63}{482.53}\right)^{3.5}$$

$$p = 361.9 \text{ Pa}$$

$$\rho = \frac{361.9}{417.63 \times 287} = 3.02 \text{ kg/m}^3$$

$$C_r = \frac{16}{3.02 \times 0.772} = 68.63 \text{ m/s}$$

Taking C_r as 62.63 m/s, the vane angle

$$\tan \alpha = \frac{C_w}{C_r}$$

$$= \frac{354.5}{68.63} = 5.17$$

i.e. $\alpha = 79°$

Mach number at vane

$$\text{M} = \left(\frac{65.68 \times 2 \times 1005}{1.4 \times 287 \times 417.63}\right)^{1/2} = 0.787$$

Design Example 4.16: The following design data apply to a double-sided centrifugal compressor:

Impeller eye root diameter:	18 cm
Impeller eye tip diameter:	31.75 cm
Mass flow:	18.5 kg/s
Impeller speed:	15500 rpm
Inlet stagnation pressure:	1.0 bar
Inlet stagnation temperature:	288 K
Axial velocity at inlet (constant):	150 m/s

Find suitable values for the impeller vane angles at root and tip of eye if the air is given 20° of prewhirl at all radii, and also find the maximum Mach number at the eye.

Solution:

At eye root, $C_{\mathrm{a}} = 150\,\mathrm{m/s}$

$$\therefore C_1 = \frac{C_a}{\cos 20°} = \frac{150}{\cos 20°} = 159.63\,\mathrm{m/s}$$

and $C_{\mathrm{w1}} = 150 \tan 20° = 54.6\,\mathrm{m/s}$

Impeller speed at eye root

$$U_{\mathrm{er}} = \frac{\pi D_{\mathrm{er}} N}{60} = \frac{\pi \times 0.18 \times 15500}{60}$$

$$U_{\mathrm{er}} = 146\,\mathrm{m/s}$$

From velocity triangle

$$\tan \beta_{\mathrm{er}} = \frac{C_a}{U_{\mathrm{er}} - C_{\mathrm{w1}}} = \frac{150}{146 - 54.6} = \frac{150}{91.4} = 1.641$$

i.e., $\beta_{\mathrm{er}} = 58.64°$

At eye tip from Fig. 4.17(b)

$$U_{\mathrm{et}} \frac{\pi D_{\mathrm{et}} N}{60} = \frac{\pi \times 0.3175 \times 15500}{60}$$

$$U_{\mathrm{et}} = 258\,\mathrm{m/s}$$

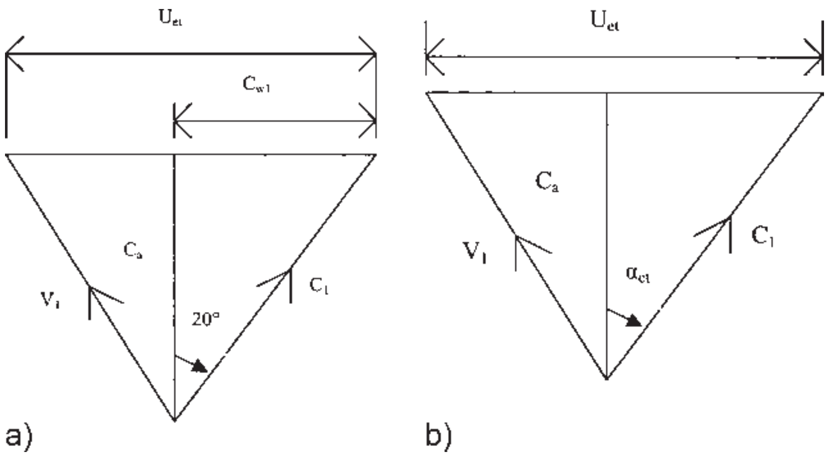

Figure 4.17 Velocity triangles at (a) eye root and (b) eye tip.

$$\tan\,\alpha_{et} = \frac{150}{258 - 54.6} = \frac{150}{203.4} = 0.7375$$

$$\text{i.e. } \alpha_{et} = 36.41°$$

Mach number will be maximum at the point where relative velocity is maximum.

Relative velocity at eye root is:

$$V_{er} = \frac{C_a}{\sin\beta_{er}} = \frac{150}{\sin 58.64°} = \frac{150}{0.8539}$$

$$V_{er} = 175.66\,\text{m/s}$$

Relative velocity at eye tip is:

$$V_{et} = \frac{C_a}{\sin\alpha_{et}} = \frac{150}{\sin 36.41°} = \frac{150}{0.5936}$$

$$V_{et} = 252.7\,\text{m/s}$$

Relative velocity at the tip is maximum.

Static temperature at inlet:

$$T_1 = T_{01} = \frac{V_{et}^2}{2C_p} = 288 - \frac{252.7^2}{2 \times 1005} = 288 - 31.77$$

$$T_1 = 256.23\,\text{K}$$

$$\text{M}_{max} = \frac{V_{et}}{\left(\gamma R T_1\right)^{1/2}} = \frac{252.7}{(1.4 \times 287 \times 256.23)^{1/2}} = \frac{252.7}{320.86}$$

$$\text{M}_{max} = 0.788$$

Design Example 4.17: In a centrifugal compressor air enters at a stagnation temperature of 288 K and stagnation pressure of 1.01 bar. The impeller has 17 radial vanes and no inlet guide vanes. The following data apply:

Mass flow rate:	2.5 kg/s
Impeller tip speed:	475 m/s
Mechanical efficiency:	96%
Absolute air velocity at diffuser exit:	90 m/s
Compressor isentropic efficiency:	84%
Absolute velocity at impeller inlet:	150 m/s

Diffuser efficiency:	82%
Axial depth of impeller:	6.5 mm
Power input factor:	1.04
γ for air:	1.4

Determine:

1. shaft power
2. stagnation and static pressure at diffuser outlet
3. radial velocity, absolute Mach number and stagnation and static pressures at the impeller exit, assume reaction ratio as 0.5, and
4. impeller efficiency and rotational speed

Solution:

1. Mechanical efficiency is

$$\eta_m = \frac{\text{Work transferred to air}}{\text{Work supplied to shaft}}$$

$$\text{or shaft power} = \frac{W}{\eta_m}$$

for vaned impeller, slip factor, by Stanitz formula is

$$\sigma = 1 - \frac{0.63\pi}{n} = 1 - \frac{0.63 \times \pi}{17}$$

$$\sigma = 0.884$$

Work input per unit mass flow

$$W = \psi\sigma U_2 C_{w2}$$

Since $C_{w1} = 0$

$$= \psi\sigma U_2^2$$

$$= 1.04 \times 0.884 \times 475^2$$

Work input for 2.5 kg/s

$$W = 1.04 \times 0.884 \times 2.5 \times 475^2$$

$$W = 518.58\text{K}$$

$$\text{Hence, Shaft Power} = \frac{518.58}{0.96} = 540.19\text{kW}$$

2. The overall pressure ratio is

$$\frac{p_{03}}{p_{01}} = \left[1 + \frac{\eta_c \psi \sigma U_2^2}{C_p T_{01}}\right]^{\gamma/(\gamma-1)}$$

$$= \left[1 + \frac{0.84 \times 1.04 \times 0.884 \times 475^2}{1005 \times 288}\right]^{3.5} = 5.2$$

Stagnation pressure at diffuser exit

$$P_{03} = p_{01} \times 5.20 = 1.01 \times 5.20$$

$$P_{03} = 5.25\,\text{bar}$$

$$\frac{p_3}{p_{03}} = \left(\frac{T_3}{T_{03}}\right)^{\gamma/\gamma-1}$$

$$W = m \times C_p(T_{03} - T_{01})$$

$$\therefore T_{03} = \frac{W}{mC_p} + T_{01} = \frac{518.58 \times 10^3}{2.5 \times 1005} + 288 = 494.4\,\text{K}$$

Static temperature at diffuser exit

$$T_3 = T_{03} - \frac{C_3^2}{2C_p} = 494.4 - \frac{90^2}{2 \times 1005}$$

$$T_3 = 490.37\,\text{K}$$

Static pressure at diffuser exit

$$p_3 = p_{03}\left(\frac{T_3}{T_{03}}\right)^{\gamma/\gamma-1} = 5.25\left(\frac{490.37}{494.4}\right)^{3.5}$$

$$p_3 = 5.10\,\text{bar}$$

3. The reaction is

$$0.5 = \frac{T_2 - T_1}{T_3 - T_1}$$

and

$$T_3 - T_1 = (T_{03} - T_{01}) + \left(\frac{C_1{}^2 - C_3{}^2}{2C_p}\right) = \frac{W}{mC_p} + \frac{150^2 - 90^2}{2 \times 1005}$$

$$= \frac{518.58 \times 10^3}{2.5 \times 1005} + 7.164 = 213.56\,K$$

Substituting

$$T_2 - T_1 = 0.5 \times 213.56$$
$$= 106.78\,\mathrm{K}$$

Now

$$T_2 = T_{01} - \frac{C_1^2}{2C_p} + (T_2 - T_1)$$
$$= 288 - 11.19 + 106.78$$
$$T_2 = 383.59\,\mathrm{K}$$

At the impeller exit

$$T_{02} = T_2 + \frac{C_2{}^2}{2C_p}$$

or

$$T_{03} = T_2 + \frac{C_2{}^2}{2C_p}(\text{Since } T_{02} = T_{03})$$

Therefore,

$$C_2{}^2 = 2C_p[(T_{03} - T_{01}) + (T_{01} - T_2)]$$
$$= 2 \times 1005(206.4 + 288 + 383.59)$$
$$C_2 = 471.94\,\mathrm{m/s}$$

Mach number at impeller outlet

$$\mathrm{M}_2 = \frac{C_2}{(1.4 \times 287 \times 383.59)^{1/2}}$$

$$\mathrm{M}_2 = 1.20$$

Radial velocity at impeller outlet

$$C_{r2}{}^2 = C_2{}^2 - C_{w2}^2$$
$$= (471.94)^2 - (0.884 \times 475)^2$$

$$C_{r2}{}^2 = 215.43 \text{ m/s}$$

Diffuser efficiency is given by

$$\eta_D = \frac{h_{3'} - h_2}{h_3 - h_2} = \frac{\text{isentropic enthalpy increase}}{\text{actual enthalpy increase}} = \frac{T_{3'} - T_2}{T_3 - T_2}$$

$$= \frac{T_2\left(\frac{T_{3'}}{T_2} - 1\right)}{T_3 - T_2} = \frac{\left[T_2\left(\frac{p_3}{p_2}\right)^{\gamma-1/\gamma} - 1\right]}{(T_3 - T_2)}$$

Therefore

$$\frac{p_3}{p_2} = \left[1 + \eta_D\left(\frac{T_3 - T_2}{T_2}\right)\right]^{3.5}$$

$$= \left(1 + \frac{0.821 \times 106.72}{383.59}\right)^{3.5}$$

$$= 2.05$$

$$\text{or } p_2 = \frac{5.10}{2.05} = 2.49 \text{ bar}$$

From isentropic $P\text{–}T$ relations

$$p_{02} = p_2\left(\frac{T_{02}}{T_2}\right)^{3.5} = 2.49\left(\frac{494.4}{383.59}\right)^{3.5}$$

$$p_{02} = 6.05 \text{ bar}$$

4. Impeller efficiency is

$$\eta_i = \frac{T_{01}\left[\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}{T_{03} - T_{01}}$$

$$= \frac{288\left[\left(\frac{6.05}{1.01}\right)^{0.286} - 1\right]}{494.4 - 288}$$

$$= 0.938$$

$$\rho_2 = \frac{p_2}{RT_2} = \frac{2.49 \times 10^5}{287 \times 383.59}$$

$$\rho_2 = 2.27 \text{ kg/m}^3$$

$$\dot{m} = \rho_2 A_2 C_{r2}$$
$$= 2\pi r_2 \rho_2 b_2$$

But

$$U_2 = \frac{\pi N D_2}{60} = \frac{\pi N \dot{m}}{\rho_2 \pi C_{r2} b_2 \times 60}$$

$$N = \frac{475 \times 2.27 \times 246.58 \times 0.0065 \times 60}{2.5}$$

$$N = 41476 \, \text{rpm}$$

PROBLEMS

4.1 The impeller tip speed of a centrifugal compressor is 450 m/s with no prewhirl. If the slip factor is 0.90 and the isentropic efficiency of the compressor is 0.86, calculate the pressure ratio, the work input per kg of air, and the power required for 25 kg/s of airflow. Assume that the compressor is operating at standard sea level and a power input factor of 1.

(4.5, 182.25 kJ/kg, 4556.3 kW)

4.2 Air with negligible velocity enters the impeller eye of a centrifugal compressor at 15°C and 1 bar. The impeller tip diameter is 0.45 m and rotates at 18,000 rpm. Find the pressure and temperature of the air at the compressor outlet. Neglect losses and assume $\gamma = 1.4$.

(5.434 bar, 467 K)

4.3 A centrifugal compressor running at 15,000 rpm, overall diameter of the impeller is 60 cm, isentropic efficiency is 0.84 and the inlet stagnation temperature at the impeller eye is 15°C. Calculate the overall pressure ratio, and neglect losses.

(6)

4.4 A centrifugal compressor that runs at 20,000 rpm has 20 radial vanes, power input factor of 1.04, and inlet temperature of air is 10°C. If the pressure ratio is 2 and the impeller tip diameter is 28 cm, calculate the isentropic efficiency of the compressor. Take $\gamma = 1.4$ (77.4%)

4.5 Derive the expression for the pressure ratio of a centrifugal compressor:

$$\frac{P_{03}}{P_{01}} = \left[1 + \frac{\eta_c \sigma \psi U_2^2}{C_p T_{01}}\right]^{\gamma/(\gamma-1)}$$

4.6 Explain the terms "slip factor" and "power input factor."

4.7 What are the three main types of centrifugal compressor impellers? Draw the exit velocity diagrams for these three types.

4.8 Explain the phenomenon of stalling, surging and choking in centrifugal compressors.

4.9 A centrifugal compressor operates with no prewhirl and is run with a tip speed of 475 the slip factor is 0.89, the work input factor is 1.03, compressor efficiency is 0.88, calculate the pressure ratio, work input per kg of air and power for 29 airflow. Assume $T_{01} = 290$ K and $C_p = 1.005$ kJ/kg K.

(5.5, 232.4 kJ/kg, 6739 kW)

4.10 A centrifugal compressor impeller rotates at 17,000 rpm and compresses 32 kg of air per second. Assume an axial entrance, impeller trip radius is 0.3 m, relative velocity of air at the impeller tip is 105 m/s at an exit angle of 80°. Find the torque and power required to drive this machine.

(4954 Nm, 8821 kW)

4.11 A single-sided centrifugal compressor designed with no prewhirl has the following dimensions and data:

Total head/pressure ratio:	3.8:1
Speed:	12,000 rpm
Inlet stagnation temperature:	293 K
Inlet stagnation pressure:	1.03 bar
Slip factor:	0.9
Power input factor:	1.03
Isentropic efficiency:	0.76
Mass flow rate:	20 kg/s

Assume an axial entrance. Calculate the overall diameter of the impeller and the power required to drive the compressor.

(0.693 m, 3610 kW)

4.12 A double-entry centrifugal compressor designed with no prewhirl has the following dimensions and data:

Impeller root diameter:	0.15 m
Impeller tip diameter:	0.30 m
Rotational speed:	15,000 rpm
Mass flow rate:	18 kg/s

Ambient temperature:	25°C
Ambient pressure:	1.03 bar
Density of air at eye inlet:	1.19 kg/m^3

Assume the axial entrance and unit is stationary. Find the inlet angles of the vane at the root and tip radii of the impeller eye and the maximum Mach number at the eye.

(α_1 at root $= 50.7°$, $\alpha_1 = 31.4°$ at tip, 0.79)

4.13 In Example 4.12, air does not enter the impeller eye in an axial direction but it is given a prewhirl of 20° (from the axial direction). The remaining values are the same. Calculate the inlet angles of the impeller vane at the root and tip of the eye.

(α_1 at root $=$ 65.5°, α_1 at tip $= 38.1°$, 0.697)

NOTATION

C	absolute velocity
r	radius
U	impeller speed
V	relative velocity
α	vane angle
σ	slip factor
ω	angular velocity
ψ	power input factor

SUFFIXES

1	inlet to rotor
2	outlet from the rotor
3	outlet from the diffuser
a	axial, ambient
r	radial
w	whirl